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ABSTRACT: The paper analyzes critically the results of previous investigators. The possibility of using the analogy between unsteady and hypersonic flow around thin bodies may be applied in the first approximation to all regions behind the compression shock, including the layer adjacent to the body. The paper shows that the contour can be determined by proper selection of entropy on the particle's trajectory forming the contour, the equation for which is found by solving the explosion problem in Lagrangian variables.

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APPLICATION OF THE EXPLOSION ANALOGY TO THE CALCULATION OF HYPERSONIC FLOWS

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After Tsien [1], Hayes [2] and A.A. Il'yushin [3] established the analogy between hypersonic flow around thin bodies and nonstationary flows in space with a smaller per unit number of measurements, the attention of a number of investigators turned to the question of how the steady-state flow relates to the gas motion resulting from a strong explosion. In earlier papers [4-8] it was considered that the gas particles in the detonation of a planar or string explosive charge move in the same way as do the particles in the flow near a blunt plate or semi-infinite cylinder set at zero angle of attack with the flow. The thicknesses of the bodies in the flow were taken to be vanishingly small and the influence of their obtuse nose sections was replaced simply by the effect of the lumped force on the surrounding medium. The analogy set up in this way allowed explaining the most general characteristic features of both phenomena but had a shortcoming in that the density at the surface of the plate turned out to be zero and the entropy infinite. /622*

In subsequent papers by Cheng [9], V.V. Sychev [10, 11] and Yakura [12], the concept of a high-entropy layer was evolved, according to which the thickness of the bodies in the flow was increased to infinity downstream but the entropy remained finite over the entire contour. In these papers it was emphasized that the flow in the high-entropy layer differs from the flow in the remaining space — specifically, the application of the hypothesis of plane sections to the calculation of this layer leads to the appearance of relatively large errors.

The results of V.V. Sychev [10, 11] and Yakura [12] are critically analyzed. The form in which they can be obtained directly from the theory of strong explosions in the papers by L.I. Sedov [13, 14] and Taylor [15] is shown. In the problem being considered this possibility means that the analogy between nonstationary flows and the hypersonic flow around thin bodies may be applied to a first approximation over all regions behind the bow compression shock, including the layer adjacent to the body contour. To determine the contour itself it is sufficient to select correctly the value of entropy on the particle trajectory forming it, the equation for which is found from the solution of the explosion problem in Lagrange variables [16].

1. We shall assume that the motion of the gas is axisymmetric, but the essential conclusions drawn below are valid for plane-parallel flows as well. We shall label the axes of the cylindrical coordinate system x and r, with the x-axis in the direction of the

*Numbers in the margin indicate pagination in the foreign text.

velocity vector of the undisturbed flow. Following [10-12] we shall consider the inverse problem, in which the shape of the compression shock is assigned [$r = r_S(x)$] and the contour of the body in the flow is found in the process of its solution. Applying the explosion analogy to the calculation of hypersonic flows, they assumed that

$$r_s = C\sqrt{x} \tag{1.1}$$

where C is an arbitrary constant.

The principal result obtained by V. V. Sychev [10] was the definition of the shape of the body beyond the point of intersection of the shock front and the center-line of the flow. The equation of the body contour, $r = r_b(x)$, was given in the form /623

$$r_b = Cx^{1/2}\left\{1 + \frac{\varkappa - 1}{\varkappa + 1}\int_1^0 G(x, \eta)\left[1 - \frac{\varkappa C^2}{(\varkappa+1)^2}\frac{G(x, \eta)H(\eta)}{x}\right]^{-1/2} d\eta\right\}^{1/2} \tag{1.2}$$

Here $\varkappa$ is the exponent in the Poisson adiabatic, the function H is the ratio of the pressure in the region of disturbed flow to the pressure behind the compression shock, and the quantity

$$G = \left[\frac{x}{H}\left(x\eta + \frac{C^2}{4}\right)^{-1}\right]^{1/\varkappa} \tag{1.3}$$

Formula (1.2) loses validity for small values of x, since the velocity field perturbations turn out to be finite and can not be described by the theory, which is based on the analogy with nonstationary flows. On the other hand, this formula becomes more accurate the larger the value of x; it is thus reasonable to simplify it by going to the limit, setting $x \to \infty$.

For the purpose as stated we shall use the relationship between the variable of integration η and the self-similar variable λ introduced by L. I. Sedov [16]. If we denote the ratio of the velocities in the zone of disturbed flow and behind the shock front of f, then [10]:

$$\eta = \exp\left(2\int_1^\lambda\left(\lambda - \frac{2}{\varkappa+1}f\right)^{-1} d\lambda\right).$$

Let us introduce another function g, the ratio between the density at an arbitrary point between the compression shock and the body to the density resulting from the strong shock compression of the gas. The relationship between the functions f and g and their first derivatives is

$$\frac{1}{g}\frac{dg}{dx} + \left(f - \frac{\varkappa+1}{2}\lambda\right)^{-1}\left[\left(\frac{df}{d\lambda} - \frac{\varkappa+1}{2}\right) + \left(\frac{f}{\lambda} + \frac{\varkappa+1}{2}\right)\right] = 0$$

as pointed out in the book by V. P. Korobeynikov, N. S. Mel'nikov and Ye. V. Ryazanov [17]. This relationship is easily converted to the form

$$\frac{d}{d\lambda}\ln\left[\lambda g\left(\frac{\varkappa+1}{2}\lambda-f\right)\right]=2\left(\lambda-\frac{2}{\varkappa+1}f\right)^{-1}$$

From this it follows that

$$\eta=\frac{2}{\varkappa-1}\lambda g\left(\frac{\varkappa+1}{2}\lambda-f\right) \tag{1.4}$$

By definition, $H(\eta) = h(\lambda)$. Using (1.4) we rewrite (1.3) as

$$G=x^{1/\varkappa}\left\{h\left[\frac{2}{\varkappa-1}\lambda g\left(\frac{\varkappa+1}{2}\lambda-f\right)x+\frac{C^2}{4}\right]\right\}^{-1/\varkappa}$$

The expression in square brackets on the right-hand side of this expression can be simplified if we use the integral of the adiabatic [17]

$$\frac{2}{\varkappa-1}\lambda gh\left(\frac{\varkappa+1}{2}\lambda-f\right)=g^{\varkappa}$$

The function G now takes on the final form /624

$$G=\left(g^{\varkappa}+\frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa} \tag{1.5}$$

On going over from the variable η to the self-similar variable λ in Eq. (1.2) for the contour of the unknown body, we have

$$r_b=Cx^{1/2}\left\{1+2\int_1^0 G(x,\lambda)\left[1-\frac{\varkappa C^2}{(\varkappa+1)^2}\frac{G(x,\lambda)h(\lambda)}{x}\right]^{-1/2}\lambda g(\lambda)\,d\lambda\right\}^{1/2} \tag{1.6}$$

With $x \to \infty$ and finite values of λ, the function $G \to g^{-1}$. When $\lambda \to 0$, $g \to 0$ and $h \to h_0 \neq 0$, as follows from the asymptotic formulas derived by L. I. Sedov [16]. Therefore as $\lambda \to 0$ and $x \to \infty$, the second of the two items in the square brackets on the right-hand side of (1.5) can turn out to be larger than the first, and with $\lambda = 0$ we have

$$G=\left(\frac{4}{C^2h_0}\right)^{1/\varkappa}x^{1/\varkappa}$$

From this it follows that the ratio $G/x \to 0$ and $x \to \infty$ and with arbitrary values of λ. Making use of this circumstance, we write the expression

$$\left[1-\frac{\varkappa C^2}{(\varkappa+1)^2}\frac{Gh}{x}\right]^{-1/2}=1+\frac{\varkappa C^2}{2(\varkappa+1)^2}\frac{Gh}{x}+\dots \tag{1.7}$$

To find the asymptotic behavior of the generatrix of the contour of the body (unknown) for large values of the x-coordinate, only the first term of the series (1.7) need be used in computing the integral on the right side of (1.6). It is easy to show that in this integral the remainder terms of the series yield a contribution of much lower order in x. Hence, as a first approximation, we find

$$r_b = Cx^{1/2}\left\{1 + 2\int_1^0 \left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa} \lambda g\, d\lambda\right\}^{1/2} \tag{1.8}$$

Here the direct expansion into series of the expressions inside the integral is no longer possible for large values of x. Hence we have

$$\int_0^1 \left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa} \lambda g\, d\lambda = \left(\int_0^\varepsilon + \int_\varepsilon^1\right)\left[\left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-1/\varkappa} \lambda g\, d\lambda\right] = J_1 + J_2$$

where the parameter ε is chosen so that on the one hand

$$g^\varkappa(\varepsilon) \gg \frac{C^2}{4}\frac{h(\varepsilon)}{x} \tag{1.9}$$

and, on the other, $\varepsilon \ll 1$. According to the condition (1.9) the expansion of the expression under the integral in J_2 is convenient. Making use of this, we find

$$J_2 = -\frac{1}{2} + \frac{\varepsilon^2}{2} - \frac{C^2}{4\varkappa}\frac{1}{x}\int_1^\varepsilon \frac{h}{g^\varkappa}\, d\lambda + \ldots$$

To evaluate the integral J_1 we reform the previous expression

$$\left(g^\varkappa + \frac{C^2}{4}\frac{h}{x}\right)^{-\frac{1}{\varkappa}} = g_0\lambda^{\frac{2}{\varkappa-1}} g^{-1}\mu^{-\frac{1}{\varkappa}}(1+m)^{-\frac{1}{\varkappa}}$$
$$m = \frac{C^2}{\mu g^\varkappa x}\left(g_0^\varkappa \lambda^{\frac{2\varkappa}{\varkappa-1}} h - g^\varkappa h_0\right), \quad \mu = g_0^\varkappa \lambda^{\frac{2\varkappa}{\varkappa-1}} + \frac{C^2}{4}\frac{h_0}{x} \tag{1.10}$$

Here the constant g_0 and h_0 are the coefficients of the first terms of the asymp- /625
totic expansions of the functions

$$g = \lambda^{\frac{2}{\varkappa-1}}\left(g_0 + g_1\lambda^{\frac{2\varkappa}{\varkappa-1}} + \ldots\right), \quad h = h_0 + h_1\lambda^{\frac{2\varkappa}{\varkappa-1}} + \ldots \tag{1.11}$$

with small values of λ. Using the asymptotic representation (1.11) it is easily shown that the quantity $m \ll 1$ for large values of x and $0 \le \lambda \le \varepsilon$.

Taking this inequality into account, we find, to a first approximation

$$J_1 = g_0 \int_0^{\varepsilon} \lambda^{\frac{\varkappa+1}{\varkappa-1}} \mu^{-\frac{1}{\varkappa}} \, d\lambda$$

If we go over from integration over λ to integration over μ in accordance with (1.10), the quantity J_1 can be calculated in final form. Retaining only the main terms, we have

$$J_1 = -\frac{\varepsilon^2}{2} + \frac{1}{2} 2^{-\frac{2(\varkappa-1)}{\varkappa}} g_0^{1-\varkappa} h_0^{\frac{\varkappa-1}{\varkappa}} C^{\frac{2(\varkappa-1)}{\varkappa}} x^{-\frac{\varkappa-1}{\varkappa}} + \ldots$$

We now collect the results obtained and substitute them into (1.8). To sum up, as $x \to \infty$, the behavior of the generatrix contour of the body in the flow will be given by the relationship

$$r_b = 2^{-\frac{\varkappa-1}{\varkappa}} g_0^{\frac{1-\varkappa}{2}} h_0^{\frac{\varkappa-1}{2\varkappa}} C^{\frac{2\varkappa-1}{\varkappa}} x^{\frac{1}{2\varkappa}} \tag{1.12}$$

To compare formula (1.12) with the analagous formula from the theory of Yakura [12] it is useful to convert to nondimensional variables by referring the values of the coordinates to the radius r_* of the shock front at the point where it intersects the axis of symmetry. From Eq. (1.1), which defines the shape of the compression shock, it follows that

$$r_s/r_* = \sqrt{2x/r_*} \quad (C = \sqrt{2r_*}) \tag{1.13}$$

Taking into account the expressions [16, 17]

$$g_0 = 2^{-\frac{2}{(\varkappa-1)(2-\varkappa)}} \varkappa^{\frac{3\varkappa-4}{(\varkappa-1)(2-\varkappa)}} (\varkappa+1)^{\frac{2}{\varkappa-1}}, \quad h_0 = 2^{-\frac{2}{2-\varkappa}} \varkappa^{\frac{2(\varkappa-1)}{2-\varkappa}} (\varkappa+1) \tag{1.14}$$

for the coefficients g_0 and h_0, we give final form to formula (1.12):

$$\frac{r_b}{r_*} = 2^{\frac{4-\varkappa}{2\varkappa(2-\varkappa)}} \varkappa^{\frac{2-\varkappa^2}{2\varkappa(2-\varkappa)}} (\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}} \left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{1.15}$$

2. Let us come back to the work of Yakura [12]. To find the shape of a body corresponding to a shock wave (1.13), a solution of the equations of gas dynamics in it was constructed by the now well-developed method of uniting the outer and inner asymptotic expansions; the essence of this method is set forth in detail in the book by Van Dyke [18]. The outer region of the flow obeyed the solution of L. I. Sedov [13, 14] and Taylor [15] for the problem of the strong explosion; the inner expansion yielded the velocity field in the layer adjacent to the body in the flow and which possesses high entropy. It was considered that perturbation theory [1-3] based on the hypothesis of plane sections and analysis with nonstationary flows are not directly applicable to the investigation of flow in a high-entropy layer.

Analysis of the formulas for the inner expansion [12] was set up from the equation

$$\frac{r_b}{r_*}=\left(\frac{\varkappa}{\varkappa+1}\right)^{1/2}\left(\frac{2}{h_0}\right)^{\frac{1}{2\varkappa}}\left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{2.1}$$

for the generating contour of the unknown body. If the second of the expressions of (1.14) is used, it is easily seen that this equation is identical with Eq. (1.15), which follows /626 from the relationship (2.1). In this way, the shape of the body in the flow corresponding to the shock (1.13) is, to a first approximation, identical according to V.V. Sychev and Yakura, although the investigative methods on which these works were based were quite different. This explains the good qualitative agreement between the results obtained by direct evaluation of the integral in (1.2) and those from formula (2.1). Some divergence between them is attributable only to the fact that the quantity x/r in [10, 12] was chosen to be relatively small. As evident from what has been said above, there are no more profound reasons [19] for this divergence.

The formulas for the inner expansion of Yakura [12] allow establishing not only the contour of the body but also the structure of the contiguous high-entropy layer. The longitudinal coordinate x and the flow function ψ are taken as independent variables, and the lateral coordinate r is given by

$$\frac{r}{r_*}=\left(\frac{\varkappa}{\varkappa+1}\right)^{1/2}\left(\frac{2}{h_0}\right)^{\frac{1}{2\varkappa}}\left(\frac{2\psi}{\rho_\infty V_\infty r_*^2}+1\right)^{\frac{\varkappa-1}{2\varkappa}}\left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{2.2}$$

Here ρ_∞ and ν_∞ denote the density and velocity in the inflow. Equation (2.1) follows from $\Psi = 0$. After using these relationships the flow function can be expressed in terms of x and r, and then we can obtain explicit expressions for the transverse component r_r of the particle velocity, the pressure p and density ρ as functions of the cylindrical coordinates. Replacing the coefficient h_0 by its value (1.14) we find, based on [12]:

$$\begin{gathered}\frac{v_r}{V_\infty}=\frac{1}{2\varkappa}\left(\frac{x}{r_*}\right)^{-1}\frac{r}{r_*},\quad \frac{p}{\rho_\infty V_\infty^2}=2^{-\frac{2}{2-\varkappa}}\varkappa^{\frac{2(\varkappa-1)}{2-\varkappa}}\left(\frac{x}{r_*}\right)^{-1}\\ \frac{\rho}{\rho_\infty}=2^{-\frac{4-\varkappa}{(\varkappa-1)(2-\varkappa)}}\varkappa^{\frac{3\varkappa-4}{(\varkappa-1)(2-\varkappa)}}(\varkappa-1)^{-1}(\varkappa+1)^{\frac{\varkappa+1}{\varkappa-1}}\left(\frac{x}{r_*}\right)^{-\frac{1}{\varkappa-1}}\left(\frac{r}{r_*}\right)^{\frac{2}{\varkappa-1}}\end{gathered} \tag{2.3}$$

Regarding the longitudinal component r_x of the particle velocity, its deviation from the inflow velocity will be

$$\frac{v_x}{V_\infty}-1=-2^{\frac{3}{\varkappa-1}}\varkappa^{\frac{2-\varkappa}{\varkappa-1}}(\varkappa+1)^{-\frac{\varkappa+1}{\varkappa-1}}\left(\frac{x}{r_*}\right)^{\frac{2-\varkappa}{\varkappa-1}}\left(\frac{r}{r_*}\right)^{-\frac{2}{\varkappa-1}} \tag{2.4}$$

We now introduce the entropy expression

$$\frac{p/(\rho_\infty V_\infty^2)}{(\rho/\rho_\infty)^\varkappa}=2(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}\frac{1}{2\Psi/(\rho_\infty U_\infty r_*^2)+1} \tag{2.5}$$

which of course depends only on the flow function Ψ. The maximum value of entropy occurs with $\Psi = 0$; it correlates with the compression of the gas in the direct compression shock.

3. We now proceed to consider the relationships from the theory of strong explosions developed by L. I. Sedov [13, 14] and Taylor [15]. Let us call time t and let E be a quantity proportional to the entropy developed in the explosion of a string charge of unit length. Then the coordinate of the compression shock is

$$r_s = \left(\frac{E}{\rho_\infty}\right)^{1/4} \sqrt{t} \qquad (3.1)$$

In applying the analogy to the calculation of hyperonic flow the quantity E was identified with the constant F_x, which is proportional to the force, and the time t was tied to the longitudinal coordinate x using the relationship [4-8]: /627

$$t = x \,/\, U_\infty \qquad (3.2)$$

Substitution into formula (3.1) yields

$$\frac{r_s}{r_*} = C_{x1}^{1/4}\left(\frac{x}{r_*}\right)^{1/2} \qquad \left(C_{x1} = \frac{F_x}{\rho_\infty U_\infty^2 r_*^2}\right) \qquad (3.3)$$

In order for (3.3) to agree with (1.13) the drag coefficient C_{x1} must be set equal to 4; this condition will be considered in detail later. As L. I. Sedov [16] has pointed out, near the explosion center the asymptotic expansions

$$v_r = \frac{1}{2\varkappa}\frac{r}{t}, \quad p = k_2\rho_\infty\left(\frac{E}{\rho_\infty}\right)^{1/2}\frac{1}{t}, \quad \rho = k_1\rho_\infty\left(\frac{E}{\rho_\infty}\right)^{-\frac{1}{2(\varkappa-1)}} t^{-\frac{1}{\varkappa-1}} r^{\frac{2}{\varkappa-1}} \qquad (3.4)$$

apply, in which the coefficients k_1 and k_2 are related to the exponent of the Poisson adiabatic as follows;

$$k_1 = 2^{-\frac{2}{(\varkappa-1)(2-\varkappa)}}\varkappa^{\frac{3\varkappa-4}{(\varkappa-1)(2-\varkappa)}}(\varkappa-1)^{-1}(\varkappa+1)^{\frac{\varkappa+1}{\varkappa-1}}, \quad k_2 = 2^{-\frac{4-\varkappa}{2-\varkappa}}\varkappa^{\frac{2(\varkappa-1)}{2-\varkappa}}$$

By changing over in the expansions (3.4) from time t to coordinate x according to (3.2) and taking into account the last two equalities, it can be verified that these expansions agree precisely with formulas (2.3) as derived by Yakura. But this agreement also implies the validity of the hypothesis of plane sections [1-3] as applied to the high-entropy layer immediately adjacent to the surface of the body in the flow. Actually, in the paper by Yakura [12] the inner expansion was in fact found as the asymptotic solution of the strong explosion with $r \to 0$, and then the asymptote found was pieced in with the complete solution of this problem. In other words, the analogy between nonstationary and hypersonic flows over thin bodies may be used to compute all regions between the compression shock front and the surface of the body.

It remains to consider the question of the shape of the body itself. It is already clear that its contour must be formed by the trajectory of one of the particles set into motion by the shock wave. To verify this we make use of the solution of the explosion problem in Lagrange variables introduced in the monograph by L. I. Sedov [16]. This solution is written in parametric form, with the dimensionless velocity $V = tv_r/r$ selected as the parameter. The value $V = 1/(2\varkappa)$ corresponds to the flow symmetry axis. Let us designate by r_0 the initial coordinate of the particle (which it had prior to the arrival of the shock front), and we have

$$V = \frac{1}{2\varkappa}(1+\Delta)$$

It is easy to check that with small values of Δ the solution [16] of the strong explosion problem in Lagrange variables has the following asymptote /628

$$\frac{r}{r_s} = 2\varkappa^{1/2}(\varkappa-1)^{-\frac{\varkappa-1}{2\varkappa}}(\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}}\Delta^{\frac{\varkappa-1}{2\varkappa}}$$

$$\frac{r_0}{r_s} = 2^{-\frac{\varkappa-1}{2-\varkappa}}\varkappa^{\frac{1}{2-\varkappa}}(\varkappa-1)^{-1/2}\Delta^{1/2}$$

After eliminating the parameter Δ and using expression (3.3) for the coordinate r_3 of the compression shock, we find

$$\frac{r}{r_*} = 2^{\frac{4-\varkappa}{2\varkappa(2-\varkappa)}}\varkappa^{\frac{2-\varkappa^2}{2\varkappa(2-\varkappa)}}(\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}}\left(\frac{r_0}{r_*}\right)^{\frac{\varkappa-1}{\varkappa}}\left(\frac{x}{r_*}\right)^{\frac{1}{2\varkappa}} \tag{3.5}$$

Formula (3.5) is identical with (1.15) with $r_0 = r_*$, from which it follows that the body contour in the flow is formed by the trajectory of a particle set into motion by the shock wave. The coordinate r_0 found is bound up with an appropriate choice of entropy on the trajectory-contour. In fact, in the problem of the strong explosion [16]:

$$\frac{p}{\rho^\varkappa} = \frac{1}{2}(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}\frac{E}{\rho_\infty{}^\varkappa}\frac{1}{r_0{}^2} \tag{3.6}$$

Changing over to dimensionless variables, we have

$$\frac{p/(\rho_\infty U_\infty{}^2)}{(\rho/\rho_\infty)^\varkappa} = 2(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}\left(\frac{r_*}{r_0}\right)^2 \tag{3.7}$$

Let us compare the entropies obtained with those specified by (2.5), in which the body in the flow corresponds to $\Phi = 0$. Both values turn out to be equal with $r_0 = r_*$. Thus, in the direct application of explosion theory to the calculation of hypersonic flow, to define the contour of the body in the flow it is only necessary that the entropy on the particle trajectories forming it be correctly assigned. This value of entropy is obtained in the shock compression of the gas in a hypersonic flow. It will be maximally permissible inasmuch as the entropy behind an oblique compression shock in the steady-state flow must be lower. On the other hand, according to the solution for the strong explosion, the particle entropies can increase without limit on approaching the symmetry axis. The maximum permissible value of entropy according to relationship (2.5) is

$$\left[\frac{p/(\rho_\infty U_\infty^2)}{(\rho/\rho_\infty)^\varkappa}\right]_{max} = 2(\varkappa-1)^\varkappa(\varkappa+1)^{-(\varkappa+1)}$$

It delineates that region of nonstationary flow which may be used for the calculation of hypersonic flow. In the remaining portion of the nonstationary flow stemming from a strong explosion of a string charge the particle entropies are too high to be found in steady-state hypersonic flow.

The streamlines near the body contour are normalized by the relationship /629

$$\frac{2\Psi}{\rho_\infty U_\infty r_*^2} = \left(\frac{r_0}{r_*}\right)^2 - 1 \tag{3.8}$$

as derived from a comparison of (2.5) and (3.7). If condition (3.8) is satisfied, Eq. (3.5) for the path of any particle goes over into (2.2), which enters into the inner expansion obtained by Yakura.

Correction (2.4) to the longitudinal components of the velocity vector is also easily found from the theory of the strong explosion. To do this, it suffices to substitute (3.4), reduced to the form (2.3), into the Bernoulli integral

$$\frac{1}{2}(v_x^2+v_r^2)+\frac{\varkappa}{\varkappa-1}\frac{p}{\rho}=\frac{1}{2}U_\infty^2$$

We note further that according to the theory of small perturbations the particle trajectories should be determined by the solution of the ordinary differential equation

$$\frac{dr}{dt} = v_r(t,r)|_{r\to 0} = \frac{1}{2\varkappa}\frac{r}{t}$$

Integration yields

$$r = At^{\frac{1}{2\varkappa}} \tag{3.9}$$

In order to determine the arbitrary constant A in proper form, we substitute the asymptotic expansions (3.4) for the pressure and density into the left-hand member of Eq. (3.6). Taking account of the relationship (3.9) between the cylindrical coordinates and time, A can be calculated from the initial position r_0 of the particle. It is easy to verify that formula (3.5) again results on changing over from t to x in accordance with (3.2). In this way, the condition of conservation of particle entropy allows establishing the correct value of the constant in the asymptotic development for its trajectory as $t \to \infty$.

4. In conclusion let us consider briefly the results of [11]. This reference also deals with the problem of finding the shape of a body generated in a stationary hypersonic flow shock of the form of (1.1) and (1.3). The Poincaré-Lighthill-Ho method of deformed coordinates, included in the book by Van Dyke [18] was used to solve this problem. The diameter d of the body's bluntness was chosen as a scale to which to relate values of the cylindrical coordinates. The equation of the shock front was written as

$$r_s/d = \varkappa_1 C_{x_2}^{1/4} \ (x/d)^{1/2}$$

This formula may be identified with the formula (1.13) used earlier if we set

$$\varkappa_1{}^4 C_{x2} = C_{x1} = 4 \tag{4.1}$$

Then the scale factor d will in no way differ from the radius r_* of curvature of the compression shock at the point of intersection with the flow symmetry axis. The equation of the contour of the unknown body, according to [11], appears as

$$\frac{r_b}{d} = 2^{-\frac{\varkappa-1}{\varkappa}} \varkappa^{\frac{1}{2}} (\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}} \varkappa_1{}^2 \varkappa_2^{-\frac{1}{2\varkappa}} C_{x2}^{\frac{2\varkappa-1}{4\varkappa}} \left(\frac{x}{d}\right)^{\frac{1}{2\varkappa}}$$

where the constant $\varkappa_2$ is expressed through $\varkappa_1$ and the coefficient h_0 used earlier by way of the formula

$$\varkappa_x = \frac{h_0 \varkappa_1{}^2}{\varkappa+1}$$

Making use of this we find at once /630

$$\frac{r_b}{d} = 2^{\frac{\varkappa^2-2\varkappa+3}{\varkappa(2-\varkappa)}} \varkappa^{\frac{2-\varkappa^2}{2\varkappa(2-\varkappa)}} (\varkappa+1)^{-\frac{\varkappa+1}{2\varkappa}} \left(\varkappa_1 C_{x2}^{\frac{1}{4}}\right)^{\frac{2\varkappa-1}{\varkappa}} \left(\frac{x}{d}\right)^{\frac{1}{2\varkappa}} \tag{4.2}$$

If we now take account of (4.1) for the drag coefficient C_{x2}, expression (4.2) goes over into (1.15) with $d = r_*$.

As seen from what has been said above, use of the methods of asymptotic expansion and deformed coordinates in the inverse problem of defining the shape of the body from the shock wave generated by it (1.13) yield the same prescription: We can use the results of the theory of the strong explosion without any change whatsoever in the entire region between the shock front and the body whose contour is formed by the trajectory of a particle having an entropy corresponding to the compression of the gas in a stationary hypersonic flow in the direct compression shock.

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